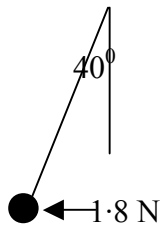
MECHANICS (C) UNIT 1 TEST PAPER 4

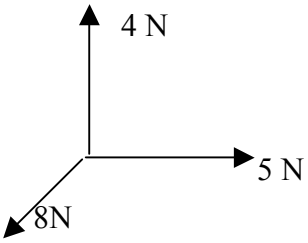
Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A small ball is attached to the end of a light inextensible string. When a horizontal force of magnitude 1·8 N is applied to the ball, it rests in equilibrium under gravity with the string taut and making an angle of 40^0 with the vertical. Calculate the mass of the ball. [4]



2. A particle P moves in a straight line. At time t seconds after passing through a fixed point O on the line, its velocity is v ms^{-1} where $v = 5 - 2\sqrt{t}$. Find
   (i) the times at which the speed of P is 1 ms^{-1}, [3]
   (ii) the magnitude of the acceleration of P when $t = 4$. [3]

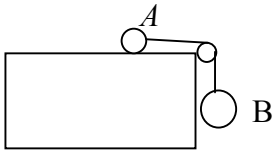
3. Forces of magnitude 4 N, 5 N and 8 N act on a particle in directions whose bearings are 000^0, 090^0 and 210^0 respectively. Find the magnitude of the resultant force and the bearing of the direction in which it acts. [7]



4. A packing case, of mass 60 kg, is standing on the floor of a lift. The mass of the lift-cage is 200 kg. The lift-cage is raised and lowered by means of a cable attached to its roof.
   In each of the following cases, find the magnitude of the force exerted by the floor of the lift-cage on the packing-case and the tension in the cable supporting the lift :
   (i) The lift is descending with constant speed. [3]
   (ii) The lift is ascending and accelerating at 1·2 ms^{-2}. [4]
   State any modelling assumptions you have made. [2]

5. Two smooth spheres X and Y, of masses x kg and y kg respectively, are free to move in a smooth straight groove in a horizontal table. X is projected with speed 6 ms^{-1} towards Y, which is stationary. After the collision X moves with speed 2 ms^{-1} and Y moves with speed 3 ms^{-1}.
   (i) Calculate the two possible values of the ratio $x : y$. [6]
   Y now strikes a vertical barrier and rebounds along the groove with speed k ms^{-1}, colliding again with X which is still moving at 2 ms^{-1}. Given that in this impact Y is brought to rest and the direction of motion of X is reversed,
   (ii) show that $k > 1{\cdot}5$. [4]

6. Two smooth spheres A and B, of masses $2m$ and m respectively, are connected by a light inextensible string which passes over a smooth fixed pulley as shown. A is initially at rest on the rough horizontal surface of a table, the coefficient of friction between A and the table being $\frac{2}{7}$. B hangs freely on the end of the vertical portion of the string. A is now projected along the table, **away from** the pulley, with speed $2{\cdot}5$ ms^{-1}.



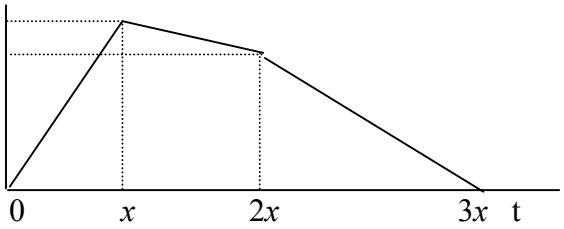
(i) State which modelling assumption ensures that the tensions in the two sections of the string can be taken to be equal. [1]

Given that A comes to rest before it reaches the edge of the table and before B hits the pulley,

(ii) find the time taken for the system to come to rest. [6]

(iii) Find the distance travelled by A before it first comes to rest. [4]

7. The diagram shows the velocity-time graph for a cyclist's journey. Each section has constant acceleration or deceleration and the three sections are of equal duration x seconds each. Given that the total distance travelled is 792 m,



(i) find the value of x and the acceleration for the first section of the journey. [5]

Another cyclist covers the same journey in three sections of equal duration, accelerating at $^1/_{11}$ ms^{-2} for the first section, travelling at constant speed for the second section and decelerating at $^1/_{11}$ ms^{-2} for the third section.

(ii) Sketch a velocity-time graph for the second cyclist. [1]

(iii) Find the time taken by this cyclist to complete the journey. [5]

(iv) Show that the maximum speeds of both cyclists are the same. [2]

MECHANICS 1 (C) TEST PAPER 4 : ANSWERS AND MARK SCHEME

1. $T \sin 40 = 1{\cdot}8$ $T = 2{\cdot}8$ $T \cos 40 = mg$ $m = 0{\cdot}219$ kg M1 A1 M1 A14

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2. (i) $5 - 2\sqrt{t} = \pm 1$ $2\sqrt{t} = 4$ or 6 $t = 4$ or $t = 9$ M1 A1 A1
   (ii) $a = -t^{-\frac{1}{2}}$ $t = 4 : a = -0{\cdot}5$ magnitude $= 0{\cdot}5$ ms^{-2} M1 A1 A1 6

3. Net force to south $= 4\sqrt{3} - 4 = 2{\cdot}928$; net force east $= 5 - 4 = 1$ M1 A1 A1
   Res. $= \sqrt{(2{\cdot}928^2 + 1^2)} = 3{\cdot}09$ N; bearing $= 90^0 + \tan^{-1}(2{\cdot}928) = 161^0$ M1 A1 M1 A17

4. (i) $T - 260g = 0$ $T = 2550$ N $R - 60g = 0$ $R = 588$ N M1 A1 A1
   (ii) $T - 260g = 1{\cdot}2 \times 260$ $T = 2548 + 312 = 2860$ N M1 A1
   $R - 60g = 1{\cdot}2 \times 60$ $R = 588 + 72 = 660$ N M1 A1

Modelled lift and case as particles, cable as light string B1 B1 9

5. (i) Momentum conserved : $6x = \pm 2x + 3y$ $4x = 3y$ or $8x = 3y$ M1 A1 A1
$x : y = 3 : 4$ or $x : y = 3 : 8$ M1 A1 A1
(ii) $2x - ky = vx$ where $v < 0$. X moving towards Y, so $x : y = 3 : 4$ M1 A1
Hence $2 - \frac{4}{3}k < 0$ $k > 1{\cdot}5$ M1 A1 10

6. (i) Smooth pulley B1
(ii) $F = ma$ for each sphere : $T + \frac{2}{7}(2mg) = 2ma$, $mg - T = ma$ B1 B1
Add : $3ma = {}^{11}/_{7}\, mg$ $a = {}^{11g}/_{21} = 5{\cdot}13$ ms^{-2} M1 A1
$v = u + at$: $0 = 2{\cdot}5 - 5{\cdot}13t$ $t = 0{\cdot}487$ s M1 A1
(iii) $v^2 = u^2 + 2as$: $0 = 2{\cdot}52 - 10{\cdot}27s$ $s = 0{\cdot}609$ m M1 A1 M1 A1 11

7. (i) Sum of areas $= 3x + 5{\cdot}5x + 2{\cdot}5x = 792$ $11x = 792$ $x = 72$ M1 A1 A1
Acc. $= 6 \div 72 = \frac{1}{12}$ ms^{-2} M1 A1
(ii) Graph sketched B1
(iii) Area under new graph $= \frac{1}{2}(3t + t)({}^{1}/_{11}t) = 792$ $4t^2 = 22 \times 79^2$ M1 A1
$t^2 = 4356$ $t = 66$ Total time $= 3t = 198$ s M1 A1 A1
(iv) $v_{max} = 66 \times {}^{1}/_{11} = 6$ ms^{-1}, as for first cyclist M1 A1 13